

Designing

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This volume of the Proceedings of the Steklov Institute of Mathematics consists mainly of studies carried out at the laboratory of approximate calculations of the Leningrad Branch of the Mathematical Institute of the Academy of Sciences of the USSR. Two studies by E. A. Volkov, whose subject lies within the scope of this volume, were carried out at the Theory of Func-
tions section of the Institute. Six papers on automatic programming are devoted to the further development of the automatic programming system designed at the LBMI under the direction of Academician L. V. Kantorovič. The authors of these papers are L. V. Kantorovič, K. V. Šahbazjan, M. M. Lebedinskiĭ, T. N. Smirnova, and V. S. Sohranskaja. Nine of the papers dealing with numerical methods have diverse subjects, and the authors are D. K. Faddeev, V. N. Kublanovskaja, V. N. Faddeeva, M. N. Jakovlev, E. A. Volkov, L. N. Dovbyš, A. P. Kubanskaja, and L. T. Savinova. The final two papers, written by V. P. Il'in and N. K. Nikol'skiĭ, are devoted to functional analysis.

Table Errata<br>503<br>Abramowitz \& Stegun 458, Aldis 459, Erdélyi, Magnus, Ober-hettinger \& Tricomi 460, Gröbner \& Hofreiter 461, Knuth 462,Kober 463, Magnus, Oberhettinger \& Soni 464, Nichol, Selfridge \&McKee 465, Vega 466.<br>Microfiche Supplement<br>Tables of Gaussian Quadrature Rules for the Calculation of Fourier Coefficients, addendum to MOC this issue, p. 245. Walter Gautschi Gaussian Quadrature Formulas for the Integration of Oscillating Functions, see review \# 24, p. 479 . . . . . . . . . Robert Piessens

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.

# Mathematics of Computation 

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tables of
gaussinn quadrature rules
FOR THE CALCULATION OF POURIER COEFFICIENTS

By

WALTER GAUTSCHI

## Writing Fourier coefficients in the form

$$
\frac{1}{2} \pi \int_{-\pi}^{\pi} f(x)_{\sin }^{\cos }(m x) d x=\int_{-1}^{1} f(\pi x)_{s}^{c} f(x) d x-\int_{-1}^{1} f(\pi x) s_{0}(x) d x
$$

where

$$
c_{m}(x)=\frac{1}{2}(1+\cos m \pi x), s_{m}(x)=\frac{1}{2}(1+\sin m \pi x),=0,1,2, \ldots
$$

are (nonnegative) veight funtions, the integrals on the right may be approximated by appropriately weighted Gaussian quadrature rules,

$$
\int_{-1}^{1} f(\pi x)_{s_{m}}^{c_{m}}(x) d x=\sum_{r=1}^{n} \lambda_{r}^{(n)} f\left(\pi \xi_{r}^{(n)}\right)
$$

Table 3 (pp. Tl-T24) relates to the weight function $c_{m}(x)$, and gives 12 D values of $\varepsilon_{r}^{(n)}, \pi \xi_{r}^{(n)}, \lambda_{r}^{(n)}$ for $n=1(1) 8,16,32$ and $m=1(1) 12$. Only the nonnegative ascissas and corresponding weights are listed, the others being obtainable from the symetry relations $\xi_{n+1-r}^{(n)}=-\xi_{r}^{(n)}$, -$\lambda_{n+1-r}^{(n)}=\lambda_{r}^{(n)}, r=1,2, \ldots, n$.

Table 4 (pp. T25-TSO) relates to the weight function $s_{\mathrm{m}}(x)$, and contains 120 values of $\varepsilon_{r}^{(n)}, \pi \xi_{r}^{(n)}, \lambda_{r}^{(n)}$ for $n=1(1) 8,16,32$ and - $-0(1) 12$.

The tables were computed in single precision floating point arithmetic on the CDC 6500 computer, using the methods described in the article "On the construction of Geussian quadrature rules from nodified moments" by the see author. (Cf., in particular, section 5(ii).)

